TWO-WHEELED SCOOTER DYNAMICS: STRAINS AND LOAD RECONSTRUCTION

Tim Hunter, Ph.D., P.E.

President, Wolf Star Technologies, LLC

SUMMARY

Dynamic field loading for FEA of consumer recreational products is not always easily replicated by traditional linear static loading methods. Wolf Star Technologies True-Load software along with unit load FEA analysis was used to determine time history loads for in service operation with dynamic loading on a Razor™ scooter. The methodology greatly improved the confidence of the design developed through simulation because the field loading was accurately reproduced. These field loads can be used in product redesign and as an input for FEA based durability calculations. In lieu of the presented technique, expensive load transducers would need to be purchased and incorporated into the test article. The test article would need to be substantially modified in order to use commercial load transducers which would ultimately change the mass and stiffness of the system. The technique presented required no changes to the underlying structure and only used simple uniaxial strain gauges. The testing incorporated an on board data collection system by Diversified Technical Systems (DTS).

Presented will be a complete evaluation of 2 wheeled Razor™ Scooter with analysis models and experimental strain measurement. This paper will show the application of the commercial software True-Load to calculate operating loads from a variety of dynamic loading events. This software leverages the concept of influence coefficients for the purpose of load reconstruction. Correlation plots of simulated strain versus measured strain will be shown. The simulated strains will be derived from the loads calculated form the load reconstruction from the measured strains. The target scooter is a Razor Scooter. This is an inexpensive children's scooter. Simulated will be X, Y, Z loads at the foot, and X, Y, Z loads and RX, RY moments at the rear wheel. The center of the steering stem is held fixed and the FEA solution uses a technique called Inertia Relief (IRL). The loading events will range from static to extreme riding over sidewalk and roadway surfaces.

TWO-WHEELED SCOOTER DYNAMICS: STRAINS AND LOAD RECONSTRUCTION

1: Introduction

The following introduction is excerpted from a paper co-authored by this author which is sited in reference [9] with minor modifications.

A structure responds to external loads (or moments) imposed on it with changes in quantities, such as stresses and strains, displacements, kinematic deformations, etc. This paper addresses the problem of measurement of time varying loads acting on a component utilizing direct strain measurement on the structure. A linear relationship between the measured strains and the applied loads is created. The relationship, i.e., the transfer function between the applied loads and the measured quantity, can be established numerically (e.g., using finite elements), mathematically, or experimentally.

Kinematic response measurements using displacement transducers and accelerometers are well established and well documented [1]. An alternative approach involves measurement of strains using strain gauges [2]. The need to measure strains, stresses or other physical quantities is apparent since these are the ultimate concern of a designer interested in ensuring structural safety. Furthermore, since the gauges are relatively inexpensive, the use of strain gauges to measure dynamic forces acting on a structure has become quite popular in structural dynamics testing [2–6]. In these works, both the normal displacement modes and the strain modes are used to describe dynamic characteristics of the structure.

While the concept of modal strain was used in the mid-1980s to describe dynamic behaviour of a structure, it was not until 1989 when Bernasconi and Ewins [3] presented a sound theoretical basis of modal stress/strain fields. The relationship between strain frequency response function and displacement frequency response function has also been explored by several authors [4–6]. While both the strain and displacement modes are intrinsic dynamic characteristics of a structure and correspond to each other, it has been noted in [6] that for sensitivity reasons, strain modal analysis is more useful in dynamic design of structures with features such as holes, grooves and cracks.

To illustrate the use of strain gauges for recovery of dynamic loads, many of the works mentioned above considered a simply supported cantilevered beam on which gauges were located in an ad hoc manner. While the gauge location on a straight cantilevered beam may be intuitive under certain loading conditions, the same cannot be said of a complex structure where a trial-and-error approach to gauge placement can result in poor load estimates. This is because the gauge may be placed at a location where it has a relatively low sensitivity to the load(s) to be estimated. Further, for multidegree of freedom force gauges, the cross-sensitivity [7] between the gauges may not be small. As a result, the strain data obtained from many of the gauges may be of little use, and the load estimates may not be precisely known.

For static loads, the influence of gauge locations and orientations on the quality of load estimates is discussed in [8]. However, in this work, it was noted that an analysis of all possible combinations of gauge placements would be too time-consuming for most problems. Consequently, only a few ad hoc groups of gauges were selected for analysis. If all possible gauge locations and orientations are not analysed, the results are not guaranteed to be optimal, which in turn, may not yield the best possible load estimates.

To overcome the shortcomings mentioned above, Dhingra, et al [9] outlines an approach for formulating and solving the gauge placement problem when the imposed loads being estimated induce vibrations in the structure, resulting in time varying dynamic strains. The accuracy of load estimates is dependent on the placement (location and orientation) of the strain gauges, and the number of strain modes retained in the analysis. A sequential exchange algorithm based approach [12,13] is used to select the optimum locations, and angular orientations of the strain gauges. This paper presents the application of this technique to a two wheeled Razor™ scooter complete with experimental measurements and comparison of simulated results to measured quantities.

2: Mathematical Foundation

Load reconstruction works on structures that behave linearly during the event of interest. The structure can undergo non-linear behavior prior to or after the event of interest. The term linear in this context means that the strain response is proportional to the applied loading. Portions of the structure may behave non-linearly. For example, local yielding near welds, bolted joints or boundary conditions may undergo non-linear strain response. Load reconstruction will continue to be effective if the nominal portions of the structure undergo linear response to the applied loading. Structures with gross yielding will not be appropriate for load reconstruction. Schematically, the concept of linearity can be illustrated as follows:

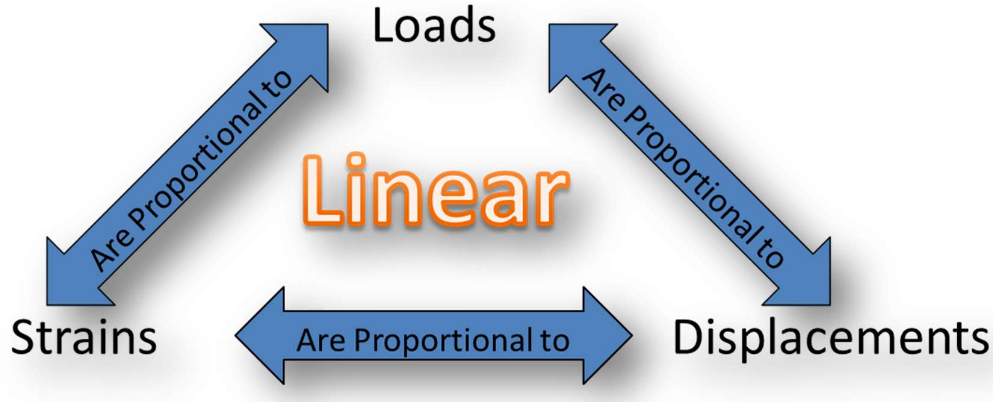

Figure 1: Linear material behaviour schematic

This linear relationship can be represented mathematically as follows:

$$F = Kx$$

Equation 1: Hooke's Law

and

$$\varepsilon C = F$$

Equation 2: Influence Coefficient Equation

Constructing a relationship for the strain equation that would work with fixed strain locations (e.g. gauges) and a series of loads cases will yield:

$$\begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \vdots & \varepsilon_{1,m} \\ \varepsilon_{2,1} & \varepsilon_{2,2} & \vdots & \varepsilon_{2,m} \\ \dots & \dots & \ddots & \vdots \\ \varepsilon_{n,1} & \varepsilon_{n,2} & \dots & \varepsilon_{n,m} \end{bmatrix} [C_{m\,x\,n}] = \begin{bmatrix} F_1 & 0 & 0 & 0 \\ 0 & F_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & F_n \end{bmatrix}$$

Equation 3: Influence Coefficient Equation Matrix Form

In the above equation the strain matrix [ε] has dimensions of n loads by m gauges. The load matrix [F] on the right hand side has dimensions of n loads by n loads. The matrix of proportionality [C] then must have dimensions of m gauges by n loads.

Each row in the strain matrix represents the strain values at a set of specific locations and orientations in the FEA model. The values in each row represent the strain response due to the corresponding load case. The columns of the strain matrix represent individual uniaxial gauge strain response. In the construct presented above, the loading matrix has been diagonalized. In general, this is not necessary, but for the developments presented here, it is convenient. Furthermore, the diagonal entries in the force matrix represent scalar multiples of the corresponding load cases. For our purposes we will set the scalar multiples to unity. This will then yield:

$$\begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \vdots & \varepsilon_{1,m} \\ \varepsilon_{2,1} & \varepsilon_{2,2} & \vdots & \varepsilon_{2,m} \\ \cdots & \cdots & \ddots & \vdots \\ \varepsilon_{n,1} & \varepsilon_{n,2} & \cdots & \varepsilon_{n,m} \end{bmatrix} [C_{m\,x\,n}] = [I]$$

Equation 4: Influence Coefficient Equation set to Unity

Then to solve for C, a simple pseudo inverse needs to be constructed

$$[C] = [\varepsilon^T \varepsilon]^{-1} \varepsilon^T$$

Equation 5: Correlation Matrix

The matrix C exists for a very large possible choices for strain gauge locations. The C matrix is optimal and most stable when the determinant of the self projected strain matrix is maximum. A sequential exchange search algorithm is deployed that looks for the gauge locations that maximize this determinant.

Once the C matrix is calculated, loading profiles can be back calculated. Given vectors of strains collected from the test structure, the loads can simply be calculated via:

$$\begin{bmatrix} \varepsilon_{t_1,1} & \varepsilon_{t_1,2} & \vdots & \varepsilon_{t_1,m} \\ \varepsilon_{t_2,1} & \varepsilon_{t_2,2} & \vdots & \varepsilon_{t_2,m} \\ \cdots & \cdots & \ddots & \vdots \\ \varepsilon_{t_{end},1} & \varepsilon_{t_{end},2} & \cdots & \varepsilon_{t_{end},m} \end{bmatrix} [C_{m\,x\,n}] = \begin{bmatrix} F_{1_{t_1}} & F_{2_{t_1}} & \cdots & F_{n_{t_1}} \\ F_{1_{t_2}} & F_{2_{t_2}} & \cdots & F_{n_{t_2}} \\ \vdots & \vdots & \vdots & \vdots \\ F_{1_{t_{end}}} & F_{2_{t_{end}}} & \cdots & F_{n_{t_{end}}} \end{bmatrix}$$

Equation 6: Time domain expansion of Forces

The strain matrix on the left hand side of the above equation represents strain gauge values (columns) at each point of time of data collection (rows). This is the strain data that has been collected from a test event. The right hand side of the equations represents a set of vectors for scaling each load case. If the individual load cases are scaled by each vector and the results are linearly superimposed, then the resulting strains at the gauge locations at the corresponding row in the test strain matrix are guaranteed to match. Furthermore, any other response in the structure that is behaving linearly will be available through this superposition.

3: Solution Procedure

Summarized next are the steps involved in the recovery of dynamic loads acting on a component which has a finite number of strain gauges located on the component to measure time varying strains.

1. Create a series of unit load cases on the FEA model that represent locations and directions of loads applied to the structure. These loads are unit loads (e.g. 1KN) and should be designed such that if they were linearly superimposed on the structure, they could approximate the operating loads. Solve the FEA model for the unit loads constructed in this step.
2. Search the structure for optimal strain gauge placement using the technique referred to in the introduction. Store the correlation matrix to disc. For the purposes of this paper, this was accomplished using Wolf Star Technologies' True-Load/Pre-Test software.
3. Place the strain gauges on the physical part and measure time histories of strain in operation.
4. Calculate the time varying loads using Equation 6.

This process can be summarized with the following diagram:

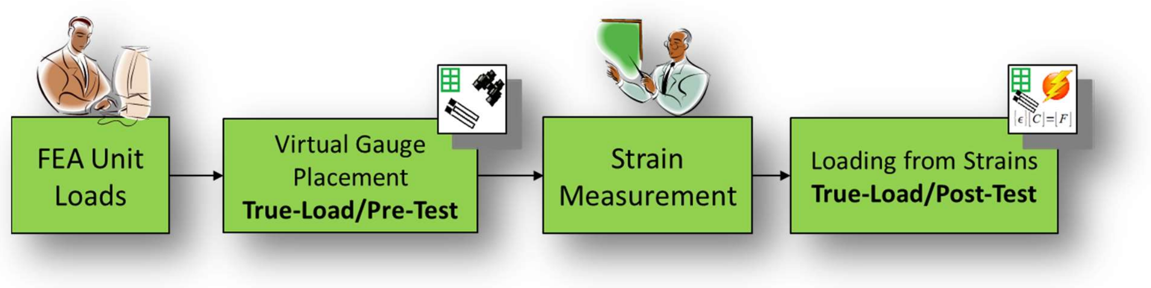

Figure 2: Load Reconstruction Process Schematic

4: The Razor™ Scooter: Problem description

Figure 3: Razor Scooter being tested

This exercise will recover the loading on Razor Scooter. The scooter was purchased from Amazon. The 3D model of the scooter was reverse engineered. A large portion of the instrumentation work was performed by Wolf Star Technologies' Intern, DeAngelo Stewart.

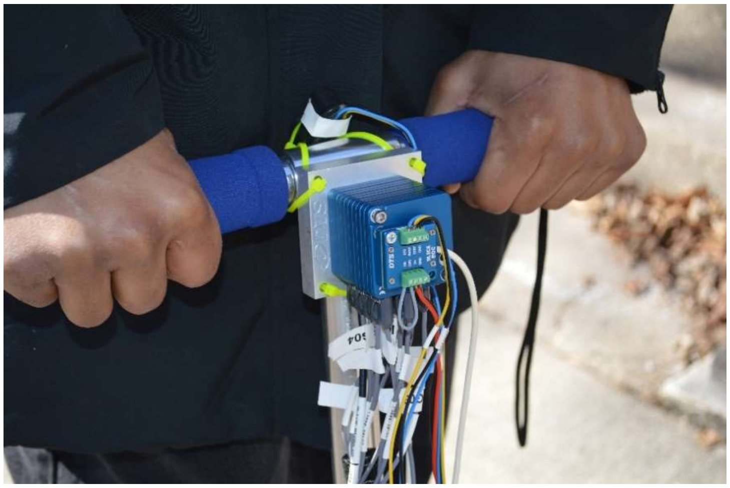

Figure 4: DTS Slice Micro DAQ

The DAQ system being used is a 12 channel DTS Slice Micro DAQ. The unit is powered by a small battery. Data is downloaded via USB cable. The strain gauges used were Micro Measurements CEA-XX-250UW-350-P2 strain gauges. These gauges are 0.250 inch gauge length gauges with pre-soldered lead wires. The lead wires are unshielded. The lead wires were trimmed short and attached to the shielded cabling of the DAQ system to minimize external electronic noise.

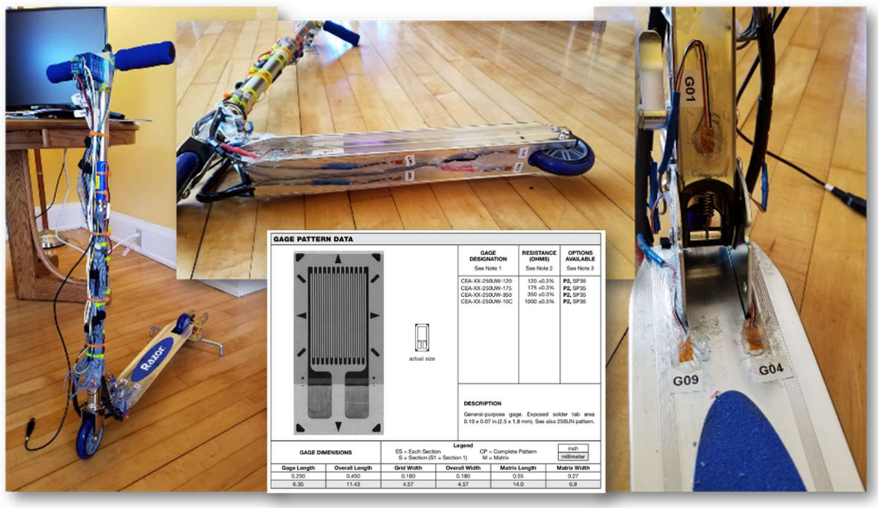

Figure 5: **Strain Gauge Placement**

5: The Razor™ Scooter: Unit Loads

The unit loads for the Razor Scooter are created in an Abaqus/CAE model. The unit loads were applied at the foot contact point (FX, FY, FZ), the rear tire patch (FY, FZ, MY) and the center of the rear axle (FX).

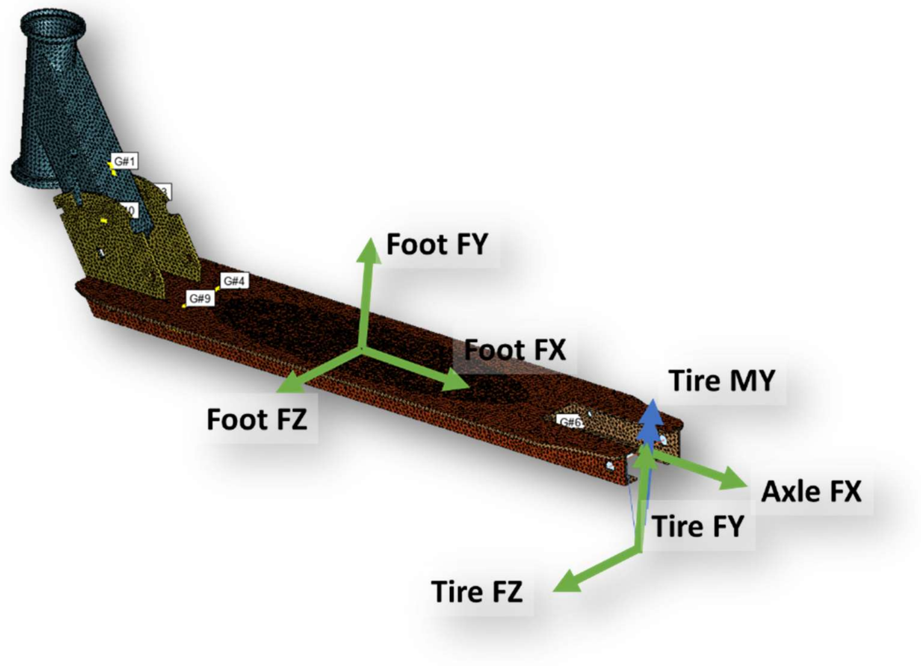

Figure 6: **FEA Unit Loads**

The bottom of the steering stem was fixed in 3DOF. The top of the steering stem was fixed in 2DOF. The 2DOF at the top are the radial DOF. A coordinate system was created at along the axis of the steering stem for the restraints. This coordinate system was also used for the inertial relief. The inertia relief was utilizing only the rotation DOF going through the center of the steering stem. The FEA solution Inertia Relief (IRL) is used to eliminate the singularities introduces by insufficient restraints.

6: The Razor™ Scooter: Pre-Test

The True-Load/Pre-Test software was used to load in the seven unit load cases and the corresponding strain results from the FEA model. The GUI from the True-Load software is shown below with the table of the unit load cases loaded.

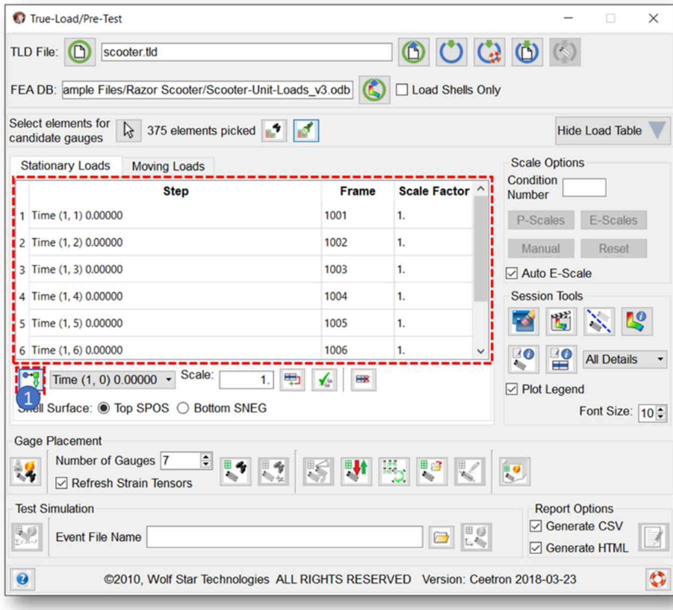

Figure 7: **Pre-Test GUI with Load Table**

The final strain gauge placement is shown below.

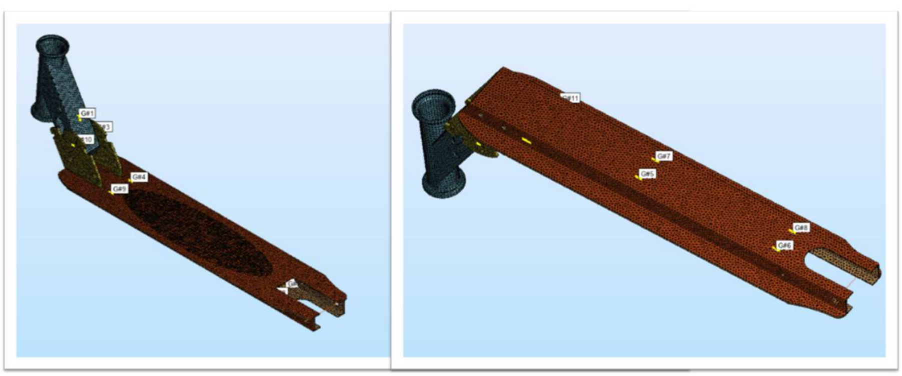

Figure 8: **Virtual Strain Gauge Placement**

An important phenomena to understand is the stability of the correlation matrix. The True-Load software provides a utility that calcuates the ideal strain for each unit load case and then applies a 5% random signal noise to the idealized strain. These strain signals are then multiplied by the correlation matrix to determine the correpsonding load response. Ideally, each load should be turned on one by one and the other loads would be turned off. The plot below shows the load sensitivty to strain noise for this configuration of gauges.

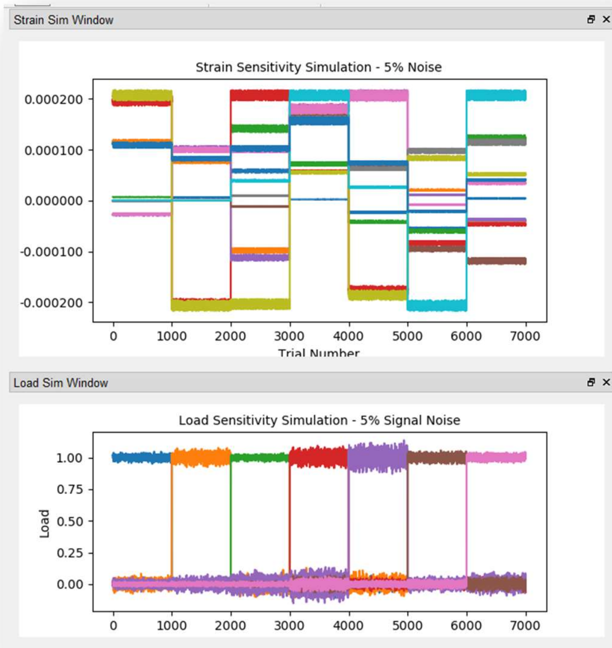

Figure 9: **Load Sensitivity to Strain Signal Noise**

This plot shows that the system of gauges chosen produces a very stable system of load reconstruction which can tolerate noise in the strain signals.

7: The Razor™ Scooter: Strain Gauge Application

A series of drawings were created which located the strain gauges on the physical structure. These drawings were then used to place the gauges on the physical part using calipers and other measurement techniques.

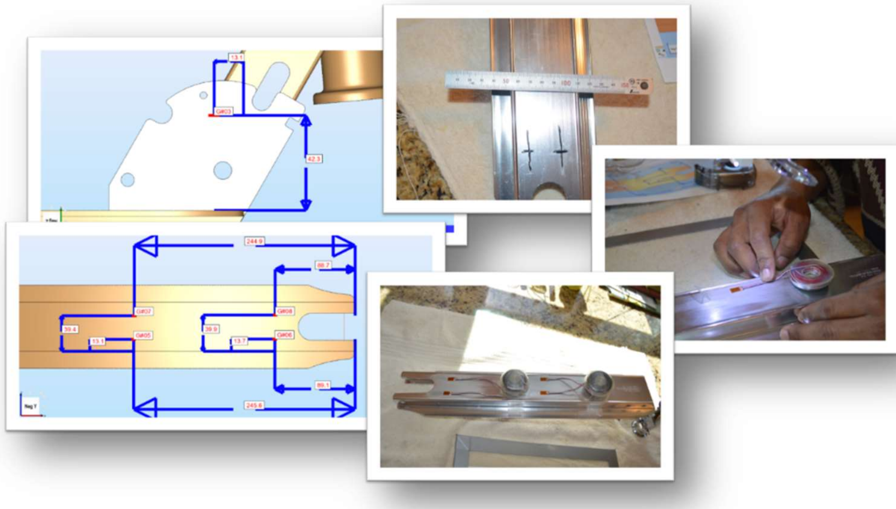

Figure 10: **Strain Gauge Installation**

8: The Razor™ Scooter: Test Data Collection

Once the scooter was fully instrumented, the strain gauges were connected to a DTS Slice Micro DAQ system. The strain data was sampled at 1000 samples per second.

Figure 11: **DAQ used for Strain Data Collection**

The data collection was performed under normal operation on a variety of surfaces. A typical trace of strain data is shown below.

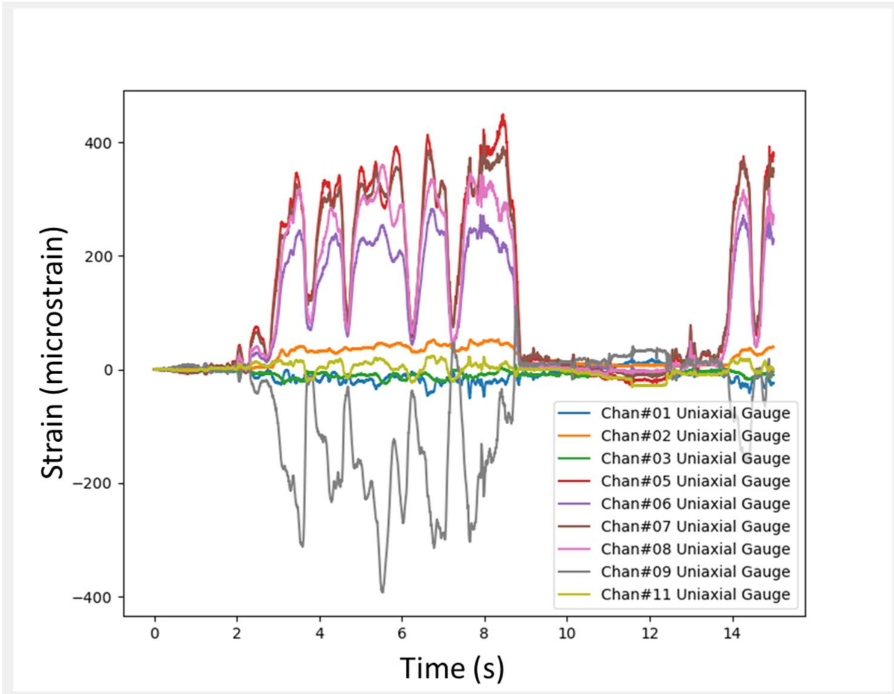

Figure 12: Typical Strain Traces from Test

9: The Razor™ Scooter: Post-Test

Once the strain data has been collected, it is processed to reconstruct the applied loading to the system. This is done by multiply the measured strain data times the correlation matrix extracted from the FEA model. The result will be a time history of loading scale factors for each of the applied loads to the scooter.

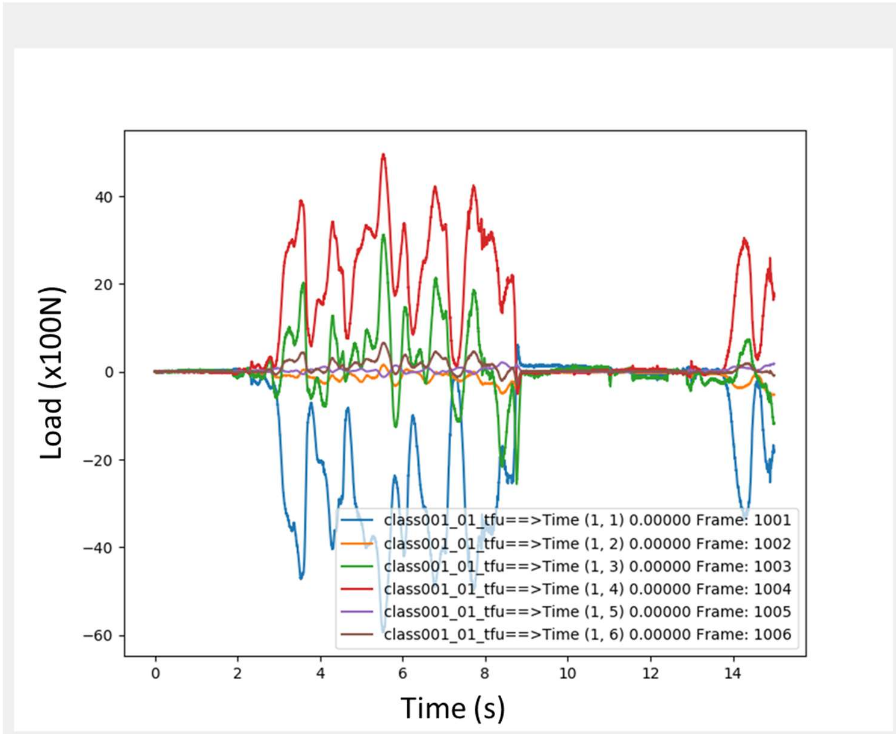

Figure 13: **Reconstructed Loads**

For this application, the True-Load/Post-Test software was used to perform this load reconstruction. In addition to the load reconstruction, several automatic post processing tasks are performed. This will produce an HTML report which contains plots of the reconstructed loads and a set of plots showing the measured strain and simulated strain from the reconstructed loads at the strain gauge locations in the FEA model. These measured / simulated strain plots are summarized in an overall plot of the simulated strain (blue) and the measured strain (green). In addition, there will be a cross plot of simulated vs measured strain. Ideally this would be a perfectly straight line on a 45 degree angle.

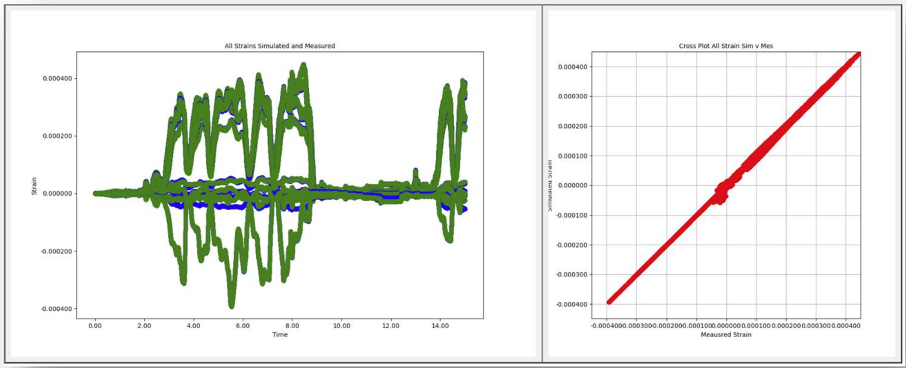

Figure 14: Overall Strain Correlation Plot

10: The Razor™ Scooter: Post-Test

Once there is confidence in the reconstructed loads, detailed post processing of the FEA model may be performed. Having a complete time history of loads it is possible to construct operating deflection shapes of the entire scooter utilizing the time history of loading and the FEA model. Below is typical plot of frame from an operating deflection shape on the scooter.

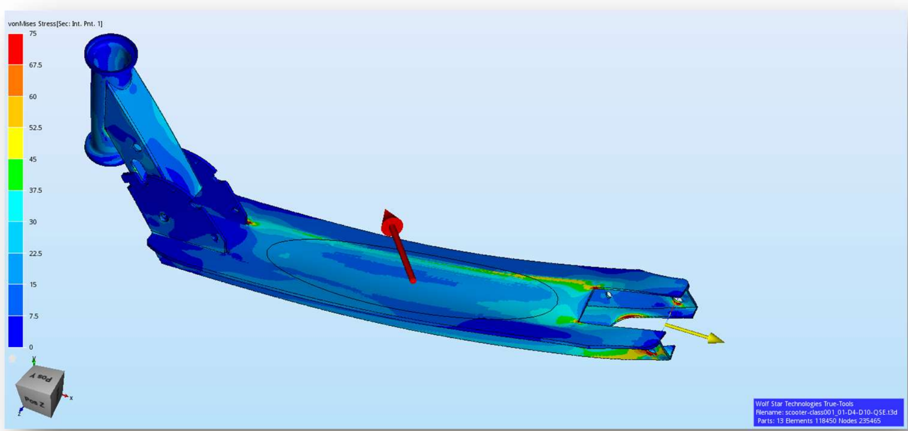

Figure 15: Typical Frame from a Reconstructed Operating Deflection Shape

11: Conclusion

It has been shown in this paper that complex / nonlinear loading on a structure can be recovered at very high accuracy. The loading DOF were sufficiently complex (FX, FY, FZ at the foot, FX at axle, FY, FZ, MY at the tire patch) to make this a non-trivial problem. If traditional load measurement techniques were to be deployed, the scooter would have been rendered inoperable. A 3 DOF load transducer could perhaps be reasonably applied at the foot location. However, extracting the 4 loading DOF on the rear wheel would have involved removing the wheel and replacing it with other load transducers yielding the device in operable.

With moderate skill and test plan processes, efficient placement of strain gauge can be placed on the structure to back calculate virtually any load conceived of by the FEA analyst. The cost for calculating these loads is two uniaxial strain gauges per loading DOF which is approximately $20. This is a highly cost effective and efficient process for determining complex loading on structures.

REFERENCES

[1] Ewins, D. J., 2000, Modal Testing: Theory, Practice, and Applications, Research Studies Press Ltd., Baldock, England.

[2] Hillary, B., and Ewins, D. J., 1984, “The Use of Strain Gages in Force Determination and Frequency Response Function,” Proceedings of the 2nd IMAC, Or- 391 lando, FL, pp. 627–634.

[3] Bernasconi, O., and Ewins, D. J., 1989, “Modal Strain Stress Fields,” J. Modal Anal., 4(2), pp. 68–76.

[4] Li, D. B., Zhuge, H., and Wang, B., 1989, “The Principles and Techniques of Experimental Strain Modal Analysis,” Proceedings of the 7th IMAC, Las Vegas, NV, pp. 1285–1289

[5] Tsang, W. F., 1990, “Use of Dynamic Strain Measurements for the Modeling of Structures,” Proceedings of the 8th IMAC, Kissimmee, FL, pp. 1246–1251.

[6] Yam, L. H., Leung, T. P., Li, D. B., and Xue, K. Z., 1996, “Theoretical and Experimental Study of Modal Strain Analysis,” J. Sound Vib., 191(2), pp. 397-398

[7] Sommerfeld, J. L., and Meyer, R. A., 1999, “Correlation and Accuracy of a Wheel Force Transducer as Developed and Tested on a Flat-Trac Tire Test Sys- 399 tem,” SAE Paper No. 1999-01-0938, Detroit, MI.

[8] Masroor, S. A., and Zachary, L. W., 1990, “Designing an All-Purpose Force Transducer,” Exp. Mech., 31(1), pp. 33–35.

[9] Dhighra, A. K, Hunter, T. G., and Gupta, D. K, 2013, “Load Recovery in Components Based on Dynamic Strain Measurements”, ASME Journal of Vibration and Acoustics, 135(5), VIB-12-1056; doi: 10.1115/1.4024384

[10] Szwedowicz, J., Senn, S. M., and Abhari, R. S., 2002, “Optimum Strain Gage Application to Bladed Assemblies,” ASME J. Turbomach., 124(4), pp. 606–613.